

BC Calc Memorization Sheet

Derivatives

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \log_b x = \frac{1}{x \ln b}$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} b^x = b^x \cdot \ln b$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \sec x = \sec x \cdot \tan x$$

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

Integration Rules

U-Substitution

$$\int f(g(x))dx \quad \text{let } u = g(x)$$

Integration by Parts

$$\int u dv = uv - \int v du$$

Decomposing into P.F.

$$\frac{1}{(cx+d)(hx+k)} = \frac{A}{(cx+d)} + \frac{B}{(hx+k)}$$

Trig Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

L'Hopital's Rule

$$\text{If } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty},$$

$$\text{then } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

Volume

Disc

$$V = \pi \int_a^b r^2 dx$$

Washer

$$V = \pi \int_a^b (R^2 - r^2) dx$$

Shell

$$V = 2\pi \int_a^b rh dx$$

Cross Section

$$V = \int_a^b A dx$$

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First Fundamental Theorem

$$\frac{d}{dx} \int_a^{g(x)} f(t) dt = f(g(x)) \cdot g'(x)$$

Second Fundamental Theorem

$$\int_a^b f(t) dt = F(b) - F(a) \quad \text{where } F'(x) = f(x)$$

Differentiation Rules

Prod.

$$\frac{d}{dx} (f \cdot g) = f'g + fg'$$

Quot.

$$\frac{d}{dx} \left(\frac{f}{g} \right) = \frac{f'g - fg'}{g^2}$$

Chain

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

Definition of Derivative

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Inv Fun Theorem

$$f(x) \quad (a,b) \quad \text{slope} = m$$

$$f^{-1}(x) \quad (b,a) \quad \text{slope} = \frac{1}{m}$$

Position, Vel, Acc

$$v(t) = \frac{d}{dt}(\text{pos}) \quad a(t) = \frac{d}{dt}(v(t))$$

$$\text{displacement} = \int_a^b v(t) dt$$

$$T.D.T. = \int_a^b |v(t)| dt$$

$$\text{speed} = |\text{vel}|$$

Alt. Series Error: error $\leq |a_{n+1}|$ (the next term)

Lagrange Error:

$$\text{error} \leq \left| \frac{f^{(n+1)}(c)(b-a)^{n+1}}{(n+1)!} \right| \quad \text{where}$$

$$|f^{(n+1)}(c)|$$

is the maximum value of $f^{(n+1)}(x)$ on $[a,b]$.

McLaurin Series to have memorized

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$$

Logistic

$$\frac{dP}{dt} = \frac{k}{M} P(M - P)$$

$$P = \frac{M}{1 + Ce^{-kt}}$$

$M =$ carrying capacity

Pt Slope Form

$$y - y_1 = m(x - x_1)$$

Taylor Series

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \dots$$

Maclaurin Series (Taylor series with $a = 0$)

Euler's Method

(x,y)	$\frac{dy}{dx}$	Δx	$\Delta y = \frac{dy}{dx} \Delta x$	(x,y)

Average Rate of Change: $AROC = \frac{f(b) - f(a)}{b - a}$ (slope between two points)

Inst. Rate of Change: $IROC = f'(c)$ (slope at a single point)

Mean Value Thm Part 1: $f'(c) = \frac{f(b) - f(a)}{b - a}$

Rolle's Thm.: if $f(a) = f(b)$, then $f'(c) = 0$

Average Value of a Function: $f_{avg} = \frac{\int_a^b f(x) dx}{b - a}$

Mean Value Thm Part 2: $f(c) = \frac{\int_a^b f(x) dx}{b - a}$

Intermediate Value Thm. A function $f(x)$ that is continuous on $[a, b]$ takes on every y -value between $f(a)$ and $f(b)$.

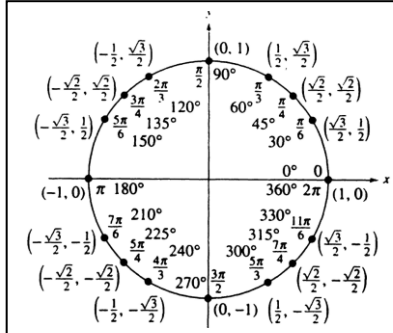
Extreme Value Thm: If $f(x)$ is continuous on $[a, b]$, then $f(x)$ must have both an absolute min and absolute max on the interval $[a, b]$.

Arc Length = $\int_a^b \underbrace{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}_{\text{cartesian}} dx$ **Arc Length** = $\int_{t_1}^{t_2} \underbrace{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}}_{\text{parametric}} dt$

Speed = $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ **T.D.T.** = $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

Polar Area = $\frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta$ **Parametric Derivatives:** $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

Polar Conversions: $r^2 = x^2 + y^2$, $x = r \cos \theta$, $y = r \sin \theta$, $\theta = \arctan \frac{y}{x}$



Area of Trapezoid
 $A = 1/2h(b_1 + b_2)$

Tests for Convergence/Divergence

n^{th} term test	div. if $\lim_{n \rightarrow \infty} a_n \neq 0$ (cannot be used to show convergence)
Geom. series test	$\sum_{n=0}^{\infty} ar^n$ $ r < 1 \rightarrow \text{conv.}$, $ r \geq 1 \rightarrow \text{div.}$, $S = \frac{a}{1-r}$
p -series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$ $p > 1 \rightarrow \text{conv.}$, $p \leq 1 \rightarrow \text{div.}$
Alternating series	decr. terms and $\lim_{n \rightarrow \infty} a_n = 0 \rightarrow \text{conv.}$
Integral test	$a_n = f(x)$ $\sum_{n=1}^{\infty} a_n$ conv. if $\int_1^{\infty} f(x) dx$ conv., $\sum_{n=1}^{\infty} a_n$ div. if $\int_1^{\infty} f(x) dx$ div.
Ratio test	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right < 1 \rightarrow \text{conv.}$, $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right > 1 \rightarrow \text{div.}$, (inconclusive if $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 1$) (works well for factorials and exponentials)
Direct Comparison	a series with terms smaller than a known convergent series also converges a series with terms larger than a known divergent series also diverges
Limit Comparison	if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n}$ is finite and positive both series converge or both diverge (use with "messy" algebraic series, usually compared to a p -series)